Determination of energy consumption according to the phases of the mineral comminution process
[Determinación del consumo energético según las fases del proceso de conminución de minerales]

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Resumen
La investigación tuvo como objetivo estudiar las ecuaciones utilizadas en el gasto energético para el área conminución de minerales y su importancia. El enfoque del artículo fue teórico, en el cual se agrupó información bibliográfica de distintas fuentes de información convincentes y fiables, mismas que se analizaron y resumieron en gráficos y tablas para entender mejor la información; así mismo se dio a conocer la importancia que estas tienen en el proceso de comercialización de minerales y las fases de producción donde las ecuaciones se utilizan; para mayor entendimiento es necesario conocer sobre la energía utilizada por los elementos en donde se da las etapas de trituración y molienda, las ecuaciones estudiadas para hallar esta energía son cuatro; la ecuación de la energía específica, el postulado de Rittinger, el postulado de Kick y el postulado de Bond, todas explicadas con sus características y los datos que consideran para ser resueltas posteriormente se presentó una comparación de sus principales diferencias además de un ejemplo práctico donde se aplicaron las tres ecuaciones y el procedimiento para resolverlas. Concluimos que la importancia de las ecuaciones de Rittinger, Kick y Bond radican en la necesidad de estas para el cálculo de costos y para dar inicio a la comercialización de minerales, la energía consumida sobre todo va a depender de las características de la roca, como tamaño, tipo de mineral valioso presente y la etapa de conminución en que se encuentren.

Palabras clave: Conminución, Comercialización, Ecuaciones, chancado, molienda.

Abstract
The objective of the research was to study the equations used in energy expenditure for the mineral comminution area and its importance. The focus of the article was theoretical, in which bibliographic information from different convincing and reliable sources of information was grouped, which were analyzed and summarized in graphs and tables to better understand the information; Likewise, the importance that these have in the mineral commercialization process and the production phases where the equations are used was made known; For a better understanding, it is necessary to know about the energy used by the elements where the crushing and grinding stages occur. The equations studied to find this energy are four; the specific energy equation, Rittinger's postulate, Kick's postulate and Bond's postulate, all explained with their characteristics and the data they consider to be resolved. Subsequently, a comparison of their main differences was presented in addition to a practical example where The three equations and the procedure to solve them were applied. We conclude that the importance of the Rittinger, Kick and Bond equations lies in the need for them to calculate costs and to begin the commercialization of minerals, the energy consumed above all will depend on the characteristics of the rock, such as size, type of valuable mineral present and the stage of comminution in which they are found.

Keywords: Conminution, Marketing, Equations, crushing, grinding.
1. Introduction

In the sphere of mining extraction, multiple stages or phases are distinguished. On this occasion, one of the most crucial within the benefit process will be analyzed. This stage takes place after extraction of the mineral, where before its commercialization, the rock containing the valuable mineral is subjected to a process called comminution. The main objective of this phase is to reduce the size of the rocks to optimize both their handling during final transportation and their subsequent treatment. Usually, the mineral is fragmented until it obtains an approximate size of 6 mm, except in particular cases. This process involves primary crushing (up to 100 mm), followed by secondary crushing (up to 10 mm) and coarse grinding. Crushing is an operation that consumes large amounts of energy and is of low energy efficiency in the subsequent comminution phase, with much of this wasted as heat, leading to the need to perform a cost-benefit analysis of these reductions to continue with the process (Palacios, 2010).

The comminution process carried out by the plant area is commonly omitted or perhaps not very well understood. If production goals are achieved, everything is considered done correctly. However, the crushing and grinding circuits spend a significant amount of energy, which can reach close to half of the operating cost of many concentrator plants, the cost of this part of production being a significant expense in the plant and that it must be taken into account prior to beginning the entire planning of the process (Metallurgist, 2023).

A panoply of regulations outlines the power required for different fractionation maneuvers. The Rittinger decree, suitable for tiny particles, with diameters less than 74 μm, advocates that the surface newly generated by pulverizing or crushing is proportional to the legitimate effort applied. The Kick norm is related to operations where the necessary force is proportional to the volumetric loss between the pre- and post-shredding particles. However, F.C. Bond, in 1951, after exhaustive analysis of multiple cases, introduced the Bond legislation. This legislation postulates that the effort invested is proportional to the magnitude of the new cleavage caused by the fragmentation of the particles, since the creation of the cleavage entails the bipartition of the rock. This opinion fills the gap left by the two previous guidelines, applicable to diameters greater than 74 μm and less than 10 cm (Yepes, 2022).

The role of fracturing and associated drilling operations is of great essentiality, because it facilitates the removal of mineral gangue, prepares the surface and particle size to concentrate valuable minerals and for subsequent processes. In terms of operating costs, the operation of these plants constitutes more than 60% of the total expenditure on mineral processing, due to their high energy consumption (Armas & Poma, 2013).

In 1867, Rittinger stated for the first time a binding proposition between the specific energy deployed and the increase in surface area experienced by particles during the spraying process. This axiom, known as its premise, or the initial rule referring to it, is like this: "The point energy invested in the fragmentation of a solid entity has a relationship of direct proportionality with the new specific surface area generated." Then, in 1874, Kirpiche and, in 1855, Kick, independently, introduced a second hypothesis, called Kick's principle. This theory states: "The force necessary to achieve parallel transformations in the dimension of geometrically analogous entities is directly proportional to the volume of said entities" (Basurto, 2016).

Subsequently, Bond's decree emerged as an antidote to the insufficiency of the elucidations provided by the Kick and Rittinger injunctions regarding the totality of the experimental events witnessed in the execution. Within the industrial domain, the urgency of a fee for the classification of materials based on their reaction to crushing procedures was foreshadowed. Therefore, in 1952, Bond formulated an empirical axiom, proclaimed as the third rule of comminution, which states: "The power deployed to contract the volume of a substance by up to 80% is in a state of
inverse proportionality with the square root of such volumetry, defining the 80% extension as the perforation of the sieve (in microns) that grants the transit of 80% in weight of the corpuscles" (Quispe, 2019).

Regarding mining, mineral extraction is carried out in gangue company and is considered economically viable when the concentration of the precious mineral justifies the investment and effort and it is possible to obtain a monetary profit from the sale of this mineral. For this to be possible, a process of separation of this gangue from the valuable mineral is necessary. Once the mineral has been extracted, the comminution process will be carried out. Communion processes are among those that generate the most economic costs and are also one of the processes that consume the most electrical energy (Mining studies of Peru, 2018).

2. Materials and Methods

This research had a qualitative research design, which searched and collected reliable information from various bibliographic sources, analyzed and interpreted the data obtained from sources such as in this case the digital platforms of the main mining companies that include the marketing process. in its production, in addition to theses, scientific articles, reports, books and relevant web pages.

It is based on theoretical assumptions and practices of the descriptive study type, the process begins with an analysis of the information on the importance of the energy used in the comminution of minerals, emphasizing the reduction of the cost of crushing and grinding and how it is spent energy in both phases, it is also important to highlight that based on the product obtained from each stage, the energy required will be different (according to its granulometry). In the final part of this first result, a summary graph of the activities commonly carried out is shown. in mine and the energy consumed by activity where this difference in consumption is visible.

Next, the energy consumed will be described according to the stages of mineral comminution, which was summarized and organized through the first table, in order to better understand the information on energy expenditure in kilowatt-hours in crushing, grinding, and its subphases based on the size of the product, such information was transferred to a curve graph simulating an example where crushing and grinding was carried out for 80% of the mineral content.

Once the two previous stages were understood, the equations were sought to find the energy consumed in the comminution process, which are the base postulates of the study; Rittinger’s postulate, Kick’s postulate and Bond’s equation each have different equations and the use of elements is consequently different, which is why in the next stage the characteristics of the equations and their main differences are described and compared, where the size of the mineral of one of the main ones, which is why a graph is included where the difference in size of the minerals is more visible thanks to the reference images of the particle size.

In the final stage, each of the equations is used to find the comminution energy previously described and characterized. This was done to show how these equations would be used in real life with practical examples.
3. Results

3.1. Importance of energy for mineral comminution
Dimensional reduction or comminution is the most onerous maneuver in mineral processing; however, the concentrates derived from this procedure, although simplified and economical, do not fully satisfy their inherent attributes. This underscores the need for a deeper and more comprehensive understanding of the fundamentals that govern this process, with a view to both theoretical and practical applications.

3.1.1. Reduction of crushing and grinding cost
These tasks are attributed to more than half of the total energy consumption, but there is still a lack of methods to gauge the effectiveness of these tasks, and the calculation of the theoretical energy necessary to achieve a specific size reduction remains an enigma.

3.1.2. Obtaining products more suitable for concentration procedures
It is known that every concentration process must be carried out within a specific size range, but the comminution and spraying maneuvers invariably produce products of varying granulometry, from the coarsest to the smallest separable particles. This compels us to adopt segmentation and concentration strategies that attempt to minimize the loss of precious substances, despite its inevitability.

In the subsequent diagram, it is observed that energy consumption reaches up to 48%, marking one of the highest percentages compared to the energy required by other activities. However, in certain scenarios it can amount to up to 70% of the energy expenditure of a mining plant, a figure that can increase when seeking to reach even smaller particle sizes. In the following phase of comminution, comminution or crushing is characterized by being an intensive task in terms of energy consumption and extremely low energy efficiency, with a lot of energy wasted as heat, which prompts an analysis to weigh the cost-benefit of such further reductions.

Figure 1. Energy consumption according to mining activity
Source: Taken from Bush 2013
3.2. Energy required according to the comminution stages

In its essence, the energy deployed during the comminution procedure is directly linked to the magnitude of the detail in the decrease in the volume of the elements in their phase. Additionally, it was specified that, in conventional crushing and pulverizing phases, the mechanical force provided to the comminution device or machinery is between one and one hundred times greater than the theoretical energy expenditure essential to generate a new surface, which implies that such energy is not integrates into the total energy transferred to the shredding installation. Up to 10% is used effectively to crush particles. Table 1

Table 1: Description of the energy consumed according to the stages of the comminution process.

<table>
<thead>
<tr>
<th>PHASE</th>
<th>SUBPHASE</th>
<th>DIMENSIONS (REFERENTIAL cm and mm)</th>
<th>INTERVAL ONLY kWh/T</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRUSHING</td>
<td>Initial</td>
<td>100 - 10</td>
<td>0.3 - 0.4</td>
</tr>
<tr>
<td></td>
<td>Secondary</td>
<td>10 - 1</td>
<td>0.3 -2</td>
</tr>
<tr>
<td></td>
<td>Tertery</td>
<td>1 - 0.5</td>
<td>0.4 -3</td>
</tr>
<tr>
<td>GRINDING</td>
<td>Primary</td>
<td>10 - 1</td>
<td>3 - 6</td>
</tr>
<tr>
<td></td>
<td>Secondary</td>
<td>1 - 0.1</td>
<td>4 – 10</td>
</tr>
</tbody>
</table>

Source: Taken from Armas & Poma, 2013

Below is a summary graph of the link that exists between energy expenditure in crushing and grinding for 80% of the main content before passing through a sieve.

![Figure 2 Particle size vs energy consumed relationship](image)

3.3. Equations to find the comminution energy

3.3.1. Rittinger's postulate (1867)

In connection with the newly formed surface, the surface extension of a specific set of particles of unified diameter is in a relationship of anti-proportionality with its own measurement.
2010). This maxim contemplates the force required for the fissure of entities solid (uniform and devoid of flaws) when the material reaches its critical distortion or cleavage level. (Tapia, 2015)

\[ E_R = K_R \left( \frac{1}{P} - \frac{1}{F} \right) \]  

(1)

Where:
- \( E_R \) = Energy expenditure in grinding (kWh/ton)
- \( K_R \) = Rittinger immutable figure
- \( P_{80} \) = Representative dimension of 80% of the particles that pass through the product
- \( F_{80} \) = Representative dimension of 80% of the particles that pass through the raw material

### 3.3.2. Kick Postulate (1885)

In 1885, Kick formulated the hypothesis that the energy required for the volumetric contraction of a matter maintains a straight proportionality correspondence with its dimensional reduction rate. This suggests that similar volumes of energy will lead to consistent geometric modifications in the magnitude of an element. Kick assumed that this, deployed in the segmentation of an element entity (uniform and devoid of defects), was limited exclusively to what was essential to deform said entity to its point of bifurcation; neglecting the additional energy required to effect such division. (Zumaran, 2012).

\[ E_K = K_K \left( \log F - \log P \right) = K_K \log \left( \frac{F}{P} \right) \]  

(2)

Where:
- \( E_K \) = Specific power in mineral comminution (kWh/ton)
- \( K_K \) = Immutable figure Kick
- \( P_{80} \) = Representative dimension of 80% of the particles that pass through the product
- \( F_{80} \) = Dimension representative of 80% of the particles that pass through the raw material

### 3.3.3. Bond equation (1951)

In 1952, Bond drew the conclusion that the power exerted to decrease the magnitude of a matter to a fraction of 80% maintains a proportional investment relationship with the square root of the representative dimension of said 80%; This last dimension being analogous to the sieving slit, in microns, which facilitates the transit of 80% by mass of the fragments. (Osorio, 2011)

\[ E_B = K_B \left[ \left( \frac{1}{F_{80}} \right)^{1/2} - \left( \frac{1}{P_{80}} \right)^{1/2} \right] \]  

(3)

Bond articulated the variable \( KB \) in harmony with the Index of Effort, \( Wi \) (denomination for the index of material labor), which symbolizes the total effort (in KWh/ton) essential to contract 1 ton. cuts matter from a hypothetically unlimited magnitude to fragments in which 80% are less than 100 microns (which represents approximately 67% of 200 meshes):

\[ W_i = K_B \left[ \left( \frac{1}{100^{1/2}} \right) - \left( \frac{1}{\alpha^{1/2}} \right) \right] = K_B \frac{1}{10} \]

\[ K_B = 10 \times W_i \]
\[ E_B = W = 10W_i \left( \frac{1}{\sqrt{P_{80}}} - \frac{1}{\sqrt{F_{80}}} \right) \]

Where:

- \( E_B \) = Specific power in mineral comminution (kWh/ton)
- \( W_i \) = Element labor factor (Kwh/short ton)
- \( P_{80} \) = Representative dimension of 80% of the particles that pass through the product
- \( F_{80} \) = Dimension representative of 80% of the particles that pass through the raw material

The \( W_i \) coefficient (Bond Stress Criterion) The \( W_i \) coefficient, also known as the Bond stress criterion, oscillates depending on the nature of the material (its toughness against compression) and the disintegration machinery used (including the lattice), of sectioning present in the classifier to complete the disintegration-categorization cycle and requires meticulous experimental determination for each specific application desired.

3.3.4. Charles equation (1957)

The forging of Bond’s tension precept was originated in 1952 by Bond himself, and by 1957, Charles introduced a comprehensive “driving force versus span” proposition that encapsulates the preceding normative trinities in the field of crushing (Rittinge et al. to the). Prior to such revelation (1937), Walker made a mathematical expression based on experience similar to that of Charles, culminating in this novel doctrine being called the “Charles Walker postulate.” Following the wisdom of these scholars, the universal guideline that concatenates the disbursement of concrete power in Comminution with the decrease in the proportion of particles, can be described by the following empirical differential equation:

\[
d\hat{E} = -C \cdot \frac{d(d)}{dn}
\]

(4)

This specific regulation that the expenditure of this particularized (d\( \hat{E} \)) essential to induce a tiny alteration of caliber \( d(d) \) in the dimension \( d \) of a fragment, is in direct relationship with that same tiny modification of measurement \( d(d) \) and in inverse proportion to the volume of the fragment raised to an empirical coefficient “n”.

The subordinate component of the equation is revealed with a negative indication, since it is the resistance power against fragmentation that the elements exhibit in the face of a foreign impulse. The elements “C” and “n” of the equation experience variations depending on both the substance and the device used in comminution. Charles made it clear that the triads of comminution guidelines (Rittinger, Kick and Bond) indicated that the “n” coefficient was affected by both the nature of the material and the comminution machinery (that is, the methodology used in the reduction operation and dimensional) (Basurto, 2016).

3.4. Comparison of the characteristics of the equations

Rittinger immersed himself in analyzing the life force for the bifurcation of perfect (homogeneous and faultless) solid entities, at their peak of alteration. Kick, on the other hand, contemplated that the power used in breaking up such entities was limited only to what was essential for their final distortion, ignoring the extra power to complete said division.

Faced with the realization that Kick and Rittinger’s hypotheses did not fully cover the pragmatic events seen and faced with the imperative need for an industrial canon for the categorization of materials based on their interaction with the crushing process, Bond, in 1952, established what would be coined as the Third Comminution Legislation:
Table 2. Description of energy expenditure according to the phases of the comminution process

<table>
<thead>
<tr>
<th>Phase</th>
<th>Subphase</th>
<th>Interval dimensions</th>
<th>Accurate power consumption (kWh/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRUSHING</td>
<td>Initial</td>
<td>F80=P80mm, P80=12-21mm</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>Secondary</td>
<td>F80=100mm, css-32mm</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>Tertiary</td>
<td>F100=45 mm, P80=12-21mm</td>
<td>1.78</td>
</tr>
</tbody>
</table>

Source: Taken from Patricio Navarro (Navarro, 2014)

Rittinger's axiom has not yet been strengthened with experimental foundations; However, in practice it has been observed that its application is more effective in the fragmentation of material with larger particles, specifically in the primary crushing phases.

On the other hand, although the principle proposed by Kick does not have robust practical support either, it has actually been proven that its effectiveness increases when grinding finer elements. Bond recognized the non-existence of rocks that are perfectly identical and uniform in shape, and postulated that the energy used in their fragmentation is proportional to the extension of the new fissures generated.

3.5. Examples where equations are used to find the energy for comminution of minerals

3.5.1. Rittinger's postulate

\[ E_R = K_R \left( \frac{1}{P} - \frac{1}{F} \right) \]
\[ P = 190 \text{ Kw} \]
\[ F_{80} = 70 \text{ mm} \]
\[ P_{80} = 20 \text{ mm} \]
\[ m^* = 15 \text{ ton} \]
\[ \frac{P}{m^*} = K_R \left( \frac{1}{20} - \frac{1}{70} \right) \]
\[ \frac{190 \text{ Kw}}{15 \text{ ton} \text{ h}} = K_R \left( \frac{1}{20} - \frac{1}{70} \right) \]
\[ 12.67 \text{ kwh ton} = K_R \left( 0.036 \frac{1}{m} \right) \]
\[ K_R = 351.9 \left( \frac{\text{KWH} \times \text{mm}}{\text{ton}} \right) \]

Energy consumed

\[ E = K_R \left( \frac{1}{P} - \frac{1}{F} \right) \]
\[ E = 351.9 \frac{\text{kwh mm}}{\text{ton}} \left( \frac{1}{30} - \frac{1}{80} \right) \]

\[ E = 3518.4 \text{ Kwh} \]

Daily energy consumed by the crusher

3.5.2. Kick Postulate

\[ E_X = K_K (\log F - \log P) = K_K \log \left( \frac{F}{P} \right) \]

\[ P = 190 \text{ Kw} \]
\[ F_{80} = 70 \text{ mm} \]
\[ P_{80} = 20 \text{ mm} \]
\[ m^* = 15 \text{ ton} \]
\[ \frac{P}{m^*} = K_K \times \log \left( \frac{F}{P} \right) \]
We calculate how much energy we are going to use to be able to reduce a mineral from 80 mm to 30 mm according to the kick criterion.

\[
\frac{190Kw}{15T_{on}} = K_{x} \cdot \log \left( \frac{70mm}{20mm} \right)
\]

\[
12.67 = K_{x} \cdot 0.544
\]

\[
\frac{12.67}{0.544} = K_{x}
\]

\[
K_{x} = 23.46 \frac{Kwh}{T_{on}}
\]

Finally replacing

\[
E_B = W = 10W_i \left[ \left( \frac{1}{P_{80}} \right) - \left( \frac{1}{F_{80}} \right) \right]
\]

Where:
- \( E_B = W \) = Power required in mineral wear (kWh/ton)
- \( W_i \) = Element stress factor (Kwh/short ton)
- \( P_{80} \) = Representative dimension of the 80% of particles transferred in the result, µm
- \( F_{80} \) = Representative dimension of 80% of particles passed through at the beginning, µm

The \( W_i \) factor (Stress Criterion) is influenced by the nature of the element (its toughness against wear) and by the wear apparatus used (including the section network in the classifier, for integral
wear circuits – categorization), and requires be calculated experimentally (in a standardized laboratory environment) for each specific use required.

\[
P = 190 \text{ Kw} \\
F_80 = 70 \text{ mm} \\
P_80 = 20 \text{ mm} \\
m_* = 15 \frac{\text{ton}}{h} \\
F_80 = 80 \text{ mm} \\
P_80 = 30 \text{ mm} \\
m_* = 20 \frac{\text{ton}}{h}
\]

\[
E = K_B \left( \frac{1}{\sqrt{P_{80}}} - \frac{1}{\sqrt{F_{80}}} \right)
\]

\[
\frac{190 \text{ Km}}{15 \frac{\text{ton}}{h}} = K_B \left( \frac{1}{\sqrt{20}} - \frac{1}{\sqrt{70}} \right) \frac{1}{\text{m}\text{m}}
\]

\[
12.67 = K_B \times 0.104\sqrt{\text{m}\text{m}}
\]

\[
K_B = 121.68 \left[ \frac{\text{Kwh}}{\text{ton}} + \sqrt{\text{m}\text{m}} \right]
\]

\[
E = 121.68 \left( \frac{1}{\sqrt{30}} - \frac{1}{\sqrt{80}} \right)
\]

\[
E = 8.61 \frac{\text{Kwh}}{\text{ton}}
\]

\[
P = 172.2 \text{ Kw} \\
E = 4133.47 \text{ Kwh}
\]

4. Conclusions

The essentiality of comminution energy focuses on the fact that it is the beginning of the commercialization phases, it is the area where the most energy is consumed compared to other mining processes, and it also represents a high cost for production.

The energy required for the two stages of comminution is; for primary crushing 0.3 to 0.4 kilowatt-hour, secondary 0.3 to 2 kilowatt-hour and tertiary 0.4 to 3 kilowatt-hour and for grinding, primary 3 to 6 kilowatt-hour and secondary 4 to 10 kilowatt-hour.

There are three equations for the calculation of this in the stages: the Rittinger postulate of 1867, the Kick postulate of 1885 and finally the Bond equation of 1951.

In the comparison of the equations, the considerations of each equation and for what size rock each equation is used, Rittinger’s postulate is used for coarse particles, however, Kick’s postulate is more used for fine particles.

We conclude regarding the practical exercises; In solving with the Rittinger equation an energy of 3518.4 kilowatt-hour was obtained, when solving with the Kick equation the result obtained was 4795.2 kilowatt-hour finally when the James Bond postulate was used the energy that was found was 4133.47 kilowatt-hour.
References


